(1) Is it possible to find two space vectors \( \vec{u} \) and \( \vec{v} \) such that \( \vec{u} \) is perpendicular to \( \langle -1, 1, 2 \rangle \), \( \vec{v} \) is parallel to \( \langle -1, 1, 2 \rangle \), and \( \vec{u} \) and \( \vec{v} \) add to \( \langle -2, 2, -4 \rangle \)?

If so, find such vectors. If not, explain why not.

Yes, it’s possible. Although the words “orthogonal decomposition” are not used in this problem, what’s being asked for is the orthogonal decomposition of \( \langle -2, 2, -4 \rangle \) relative to the direction given by \( \langle -1, 1, 2 \rangle \). More specifically,

\[
\vec{v} = \text{proj}_{\langle -1, 1, 2 \rangle} \langle -2, 2, -4 \rangle = \frac{\langle -2, 2, -4 \rangle \cdot \langle -1, 1, 2 \rangle}{\langle -1, 1, 2 \rangle \cdot \langle -1, 1, 2 \rangle} \langle -1, 1, 2 \rangle = -\frac{4}{6} \langle -1, 1, 2 \rangle = \langle \frac{2}{3}, \frac{-2}{3}, \frac{-4}{3} \rangle,
\]

\[
\vec{u} = \langle -2, 2, -4 \rangle - \vec{v} = \langle \frac{-8}{3}, \frac{8}{3}, \frac{-8}{3} \rangle
\]

solves the problem.

(2) Suppose the unit vector with the same direction as \( \langle 3, -2, 6 \rangle \) is represented as a directed line segment \( \overrightarrow{PQ} \) that ends at \( Q(-2, 1, 5) \). What are the coordinates of point \( P \)?

Let’s call the unit vector of this problem \( \vec{u} \).

Then \( \vec{u} = \frac{\langle 3, -2, 6 \rangle}{\|\langle 3, -2, 6 \rangle\|} = \frac{\langle 3, -2, 6 \rangle}{\sqrt{3^2 + (-2)^2 + 6^2}} = \langle \frac{3}{7}, \frac{-2}{7}, \frac{6}{7} \rangle. \) To represent \( \vec{u} \) as a directed line segment \( \overrightarrow{PQ} \) we need \( \vec{u} = Q - P, \) i.e. we need \( P = Q - \vec{u} = (-2, 1, 5) - \langle \frac{3}{7}, \frac{-2}{7}, \frac{6}{7} \rangle = \langle \frac{-17}{7}, \frac{9}{7}, \frac{29}{7} \rangle. \)