MA-341-1: Differential Geometry - Spring 2025 Syllabus

Department of Mathematics, Albert Nerken School of Engineering The Cooper Union for the Advancement of Science and Art

Instructor Robert Smyth, Professor, Mathematics.

See faculty.cooper.edu/smyth for office hours and contact information.

Class Hours Tuesdays 6-8:50 PM (Room 104)

Catalog Description

Theory of curves and surfaces, curvature, torsion, mean and Gaussian curvatures, length, area, geodesics, 1st and 2nd quadratic forms, conformal mapping, minimal surfaces, tensor formulation and applications. 3 credits. Prerequisites: Ma 223

Required Textbook O'Neill, B., *Elementary Differential Geometry*, revised 2^{nd} edition, Elsevier (2006), ISBN-13: 978-0120887354

Supplementary Texts Pressley, A., *Elementary Differential Geometry*, 2nd edition, Springer-Verlag (2010), ISBN-13: 978-1848828902

Do Carmo, M., Differential Geometry of Curves and Surfaces, revised and updated 2^{nd} edition, Dover (2016), ISBN-13: 978-0486806990

Course Objectives Demonstrate proficiency performing computations and proving theorems with the fundamental tools and techniques of elementary differential geometry.

Course Topics

- 1. Review of planar and space curves, directional derivatives. The tangent map.
- 2. Review of curvature and the Frenet formulas. Introduction to differential forms.
- 3. Covariant derivatives. Frame fields.
- 4. Connection forms. Structural equations.
- 5. Isometries of Euclidean space. Congruence of curves.
- 6. Patch computations on surfaces.
- 7. Functions, tangent vectors, and forms on surfaces.
- 8. Mappings of surfaces. Manifolds.
- 9. The Shape Operator. Normal and Gaussian curvature.
- 10. Principle curves, asymptotic curves, geodesics. Surfaces of revolution.
- 11. The Fundamental Equations.
- 12. Local isometries. Gauss's Theorema Egregium.
- 13. Integration and orientation.
- 14. Gauss-Bonnet Theorem.

Assessment

Your term grade will be determined by homework (15%), two midterms (25% each) and a cumulative final exam (35%). Homework problems and due dates are listed online at faculty.cooper.edu/smyth/ma341/hw-2025.pdf. The exams will test your familiarity with the principles of the subject with routine problems and your ability to extrapolate creatively from these principles on challenging problems. See **Grades of Record** at cooper.edu/engineering/curriculum/academic-standards-regulations for the letter grades used for this course and their proper interpretation. The exam schedule is posted at http://faculty.cooper.edu/smyth/ma341/ma341.htm.

Homework Policies

You may work individually or in groups of at most four students. A group must make a single joint submission with all team members listed at the top of the front page. In the

case of a group submission all members will receive the same grade for the assignment. You should make sure you understand all the solutions your team is submitting, even if other team members came up with them. Any assistance you (or your group) receives and all sources you use in preparing your homework assignments must be properly credited in writing on your submission. Groups may be re-formed on an assignment-by-assignment basis. Homework assignments must be submitted at the beginning of the class period on the assignment due date. Each assignment includes all problems from the list at faculty.cooper.edu/smyth/ma341/hw-2025.pdf not previously submitted up to and including those pertaining to the last section completed as of the end of the week before the assignment due date. Select problems from each assignment will be graded carefully and each assignment will be assessed for completeness. Credit will not be awarded for late submissions. However your lowest homework grade will be dropped.

Exam Policies All exams are closed book / closed notebook exams. You may use a basic scientific calculator, but no graphing or programmable calculators, computers, cellphones, books, notebooks, or other resources may be used. Bring a pencil or pen on the day of the exam. Paper will be provided. Exams are timed. Your score may be reduced if you do not stop working on your exam after time has been called. If you choose to leave the exam room during the exam period you will not be permitted to resume working on the exam after returning. Registered students must sit for the course examinations on the scheduled day and time. Failure to do so will generally result in forfeiture of the percentage credit allotted to the missed exam. Students who have medical excuses for missing an exam should contact the Dean of Students promptly. Failure to register a request for a medically excused absence in a timely manner with the Dean of Students may complicate and potentially invalidate the request. Any student requesting a medically excused absence must provide the Dean of Students with documentation from a medical provider justifying the absence. The Dean of Students will inform me when an absence is due to a valid medical issue/condition so that the absence can be considered excused. In the event of an excused absence, the student must schedule an oral makeup exam with me as soon as is possible.

ADA Accommodations If you believe you are entitled to an accommodation through the Americans with Disabilities Act you must self-identify to the Office of Student Affairs, and meet with me during the first or second week of the term to discuss arrangements for meeting your accommodation. You should either come to this meeting with a copy of your Accommodation Verification Letter from the Office of Student Affairs, or email a copy to me *prior* to the meeting. Cooper Union has limited resources and extra time may be required for accommodation arrangement to be feasible. If you are entitled to an accommodation on exams, you must confirm exam accommodation requests with me (by email) promptly after our meeting discussing your Accommodation Verification Letter. Students will not be afforded any special accommodations retroactively, i.e., for academic work completed prior to disclosure of the disability to me. Support services for students are described here.