We continue at 8:07 PM

**Def:** Suppose each of the fcn's $f_1, f_2, \dots, f_n$ possess (at least) $n-1$ derivatives (on $I$). Then the Wronskian of these fcn's (on $I$) is

$$W[f_1,\dots,f_n](x) = \begin{vmatrix} f_1 & \cdots & f_n \\ f_1' & \cdots & f_n' \\ \vdots & & \\ f^{(n-1)} & \cdots & f_n^{(n-1)} \end{vmatrix}.$$

**Ex:** If $f_1(x) = 6x^2-2x+4$, $f_2(x) = 3x^2+2$, & $f_3(x)=x$, then

$$W[f_1,f_2,f_3](x) = \begin{vmatrix} 6x^2-2x+4 & 3x^2+2 & x \\ 12x-2 & 6x & 1 \\ 12 & 6 & 0 \end{vmatrix}$$

$$= (6x^2-2x+4)(-6) - (3x^2+2)(-12) + (x)(-12)$$

$$= 0$$

**Thm 4.3** If $f_1,\dots,f_n$ possesses (at least) $n-1$ derivatives and are lin. dep on $I$ then $W[f_1,\dots,f_n](x) \equiv 0$ on $I$.

**Ex:** Let $f_1(x) = \cos x$ and $f_2(x) = \sin x$.

$$W[f_1,f_2](x) = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x - (-\sin^2 x) = 1 \neq 0.$$

It follows that $\{\cos x, \sin x\}$ can not be lin. dep., i.e. $\{\cos x, \sin x\}$ is lin. ind (on any interval).

**Thm** If $y_1,\dots,y_n$ all solve an $n^{th}$ order LHODE on $I$ then they form a lin ind set on $I$ iff $W[y_1,\dots,y_n]$ is never $0$ on $I$.

**Def:** Any set of $n$ linearly independent sol's of the $n^{th}$ order LHODE $L[y(x)] = 0$ on $I$ is called a fundamental set of soln's (on $I$).

**Ex:** The 2 fcn. set $\{\cos x, \sin x\}$ solves the $2^{nd}$ order LHODE $y'' + y = 0$ and is thus a fundamental set of soln's.

$$y'' + y = 0$$
$$\Longleftrightarrow$$
$$(D^2+1)[y] = 0$$

$$(6D^2 - D + 2)[y] = 0$$
$$\Longleftrightarrow$$
$$6y'' - y' + 2y = 0$$

**Thm:** Suppose $\{y_1,\dots,y_n\}$ is a fundamental set of soln's of the $n^{th}$ order LHODE $L[y(x)] = 0$ on $I$. Then the $n$ parameter family $Y(x) = c_1y_1 + \cdots + c_ny_n$ is the general soln. on $I$.

**Ex:** $Y(x) = c_1\cos x + c_2 \sin x$ is the general soln. (on $(-\infty,\infty)$) ~~for the~~ ODE $y''+y=0$.

**Existence Thm** Suppose $L = a_nD^n + \dots + a_1 D + a_0$, const (or at least continuous w/ $a_n$ never 0). Then $\exists$ a fundamental set of soln's for the LHODE $L[y] = 0$.

Non-homogeneous (linear) eqn's:

Recall that any soln. of an ODE which is free of arbitrary parameters is called a particular soln. for the eqn.

**Ex:** $y_p = x+1$ is a particular soln (on $(-\infty,\infty)$) of the (non hom) ODE

$$(D-1)[y] = -x.$$

**Thm** Let $L$ be an $n^{th}$ order linear differential operator. If $Y_c$ is the general soln. of the LHODE $L[y] = 0$ on $I$, and $y_p$ is any soln. of the (non-hom) ODE $L[y] = g(x)$ on $I$, then cont.

$$Y(x) = (Y_c(x) + y_p(x))$$ is the general soln. on $I$ of the (non-hom) ODE $L[y] = g(x)$.