$$\frac{dT}{dt} = K(T-5) \qquad \int \frac{dT}{T-5} = \int K\,dt$$

K ← ??

$T(1) = 55$
$T(5) = 30$

$$\ln(T-5) = Kt + C$$

(1) $\ln 50 = K + C$
(2) $\ln 25 = 5K + C$

solve for K & C !

Eqn (2) − (1): $\ln\frac{25}{50} = 4K \Rightarrow K = -\frac{1}{4}\ln 2 \approx -0.173$

From (1): $C = \ln 50 + \frac{1}{4}\ln 2 = \ln(50\cdot 2^{1/4}) \approx 4.085$

$$\ln(T-5) = -.173t + 4.085$$

At time $t=0$:

$$\ln(T-5) = 4.085$$
$$T-5 = e^{4.085}$$
$$T(0) \approx 5 + e^{4.085} \approx 64.44\ (^{\circ}F)$$

indoor temp

3.2: Nonlinear models

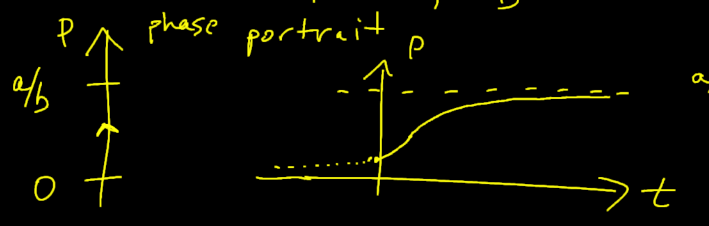
Logistic eqn. $\quad \frac{dP}{dt} = P(a - bP)$

constants

b usually much smaller than a

Critical pts: $P = 0,\ \frac{a}{b}$

phase portrait



$$\int \frac{dP}{P(a-bP)} = \int dt$$

Try partial fractions: $\frac{1}{P(a-bP)} = \frac{C}{P} + \frac{D}{a-bP}$

$$1 = (a-bP)C + PD$$

$P=0$: $\quad 1 = aC \Rightarrow C = 1/a$

$P=\frac{a}{b}$: $\quad 1 = \frac{a}{b}D \Rightarrow D = b/a = (-b)\left(\frac{-1}{a}\right)$

$$\frac{1}{a}\ln P - \frac{1}{a}\ln(a-bP) = t + C$$
$$\ln P - \ln(a-bP) = at + C$$
$$\ln\frac{P}{a-bP} = at + C$$
$$\frac{P}{a-bP} = Ce^{at}$$
$$P = (a-bP)Ce^{at}$$
$$P(1 + bCe^{at}) = aCe^{at}$$
$$P = \frac{aCe^{at}}{1 + bCe^{at}} \quad \text{(can be simplified)}$$
$$= \frac{aC}{e^{-at} + bC}$$
$$= \frac{a}{\tilde{C}e^{-at} + b} \longrightarrow \frac{a}{b} \quad (\text{as } t \to \infty)$$

decays away