MA111 (Calculus I) - Worksheet on Fundamentals from Precalculus

In addition to the A.1 homework from week 1, this worksheet is intended to give you some practice with fundamental techniques from pre-calculus. Please assure that you are able to solve all of the exercises quickly and thoroughly. You are welcome to stop by for office hours if you have any questions.

Sec. 1: Determine all real number values of x satisfying the equation:

(a) $x^2 - x - 2 = 0$;
(b) $x^2 - 9 = 0$;
(c) $x^2 = 6x - 9$;
(d) $-2x^2 + 3 = -5x$;
(e) $x^2 - 2x + 2 = 0$;
(f) $x^2 + 8x + 16 = 0$;
(g) $x^2 + 4x + 13 = 0$;
(h) $x^2 - 2x + 5 = 0$;
(i) $9x^2 - 4 = 0$;
(j) $27x^3 - 8 = 0$;
(k) $8x^6 + 64 = 0$;
(l) $4x^4 - 4x^2 + 1 = 0$.

Sec. 2: Solve the following inequalities and express the solution in interval notation:

(a) $x + 3 < 5$;
(b) $-3x + 2 < 4$;
(c) $-1 < 3 - 3x \leq 6$;
(d) $x^2 - 3x + 4 > 0$;
(e) $(x-1)(x+2)(x-3) \geq 0$;
(f) $x^3 - 3x^2 - 4x < 0$;
(g) $32x^{10} > 1$;
(h) $x^{2/3} - 4 \leq 0$;
(i) $(1/3)x^{3/2} - 2 > 7$;
(j) $\dfrac{x(x+2)}{x-2} \leq 0$;
(k) $\dfrac{(1-x)(x+2)}{x(x+1)} > 0$;
(l) $\dfrac{(1-x)(x+3)}{(x+1)(2-x)} \leq 0$;
(m) $-2 \leq \dfrac{1}{x}$;
(n) $\dfrac{2}{x-1} \geq \dfrac{3}{x+2}$;
(o) $\dfrac{2}{x-1} - \dfrac{x}{x+1} \leq -1$;
(p) $|3 - x| \geq 2$;
(q) $\left|\dfrac{3}{2x+1}\right| < 1$;
(r) $|x^2 - 4| \leq 1$.

Sec. 3: Find the center and radius of the circle:

(a) $x^2 - 2x + y^2 = 3$;
(b) $x^2 + y^2 + 4y = -3$;
(c) $x^2 + 2x + y^2 - 4y = -3$;
(d) $x^2 - 2x + y^2 + 4y = 4$;
(e) $x^2 - 4x + y^2 - 2y - 4 = 0$;
(f) $x^2 + 4x + y^2 + 6y + 9 = 0$.

Sec. 4: Indicate on an xy-plane those points (x, y) for which the statement holds:

(a) $y = -3$;
(b) $x < -2$;
(c) $x \leq -3$ and $y > 4$;
(d) $2 \leq |x|$;
(e) $-3 < x \leq 1$ and $-1 \leq y \leq 2$;
(f) $|x - 2| \leq 4$ and $y + 3 < 7$.

Sec. 5: Sketch the indicated region in the xy-plane:

(a) $\{(x, y) \mid x^2 + y^2 \leq 1\}$;

(b) $\{(x, y) \mid (x-1)^2 + y^2 > 2\}$;

(c) $\{(x, y) \mid 1 < x^2 + y^2 < 4\}$;

(d) $\{(x, y) \mid 4 \leq (x-1)^2 + (y-1)^2 \leq 9\}$;

(e) $\{(x, y) \mid x^2 + y^2 \leq 4 \text{ and } y \geq x\}$;

(f) $\{(x, y) \mid x^2 + y^2 \geq 4 \text{ and } y \leq x\}$;

(g) $\{(x, y) \mid |x| + |y| = 1\}$;

(h) $\{(x, y) \mid |x| + |y| \leq 4\}$;

(i) $\{(x, y) \mid |x-1| + |y+2| \leq 2\}$;

(j) $\{(x, y) \mid |x+3| > |y|\}$.

Sec. 6: Find the quotient polynomial $Q(x)$ and remainder polynomial $R(x)$ when the polynomial $P(x)$ is divided by the polynomial $D(x)$:

(a) $P(x) = 2x^2 + 3x - 3$ $\quad D(x) = x - 2$;

(b) $P(x) = 3x^2 - 3x - 2$ $\quad D(x) = x + 1$;

(c) $P(x) = x^3 + x^2 - 2$ $\quad D(x) = x - 1$;

(d) $P(x) = 3x^4 + 2x^3 - x + 2$ $\quad D(x) = x^2 + 2x - 1$;

(e) $P(x) = 3x^4 - 4x^3 + 2x^2 + x + 1$ $\quad D(x) = x^2 + x + 1$;

(f) $P(x) = 2x^5 + 3x^4 - 2x^3 + x^2 + x - 1$ $\quad D(x) = x^4 + x^3 - 2x - 1$.

Sec. 7: Find all rational zeros of the polynomial. Then determine any irrational zeros, and factor the polynomial completely into a product of linear and irreducible quadratic factors (only real number coefficients for the factors in this factorization!). Lastly, provide a sign chart for the polynomial.

(a) $x^3 - 3x^2 + 4$;

(b) $3x^3 - 8x^2 - 5x + 6$;

(c) $2x^4 + x^3 - 5x^2 + 2x$;

(d) $3x^4 - 11x^3 + 5x^2 + 3x$;

(e) $2x^5 + x^4 - 12x^3 + 10x^2 + 2x - 3$;

(f) $x^4 - 4x^3 + 3x^2 + 4x - 4$;

(g) $x^3 + 2x^2 - 4x + 1$;

(h) $x^4 - x^3 - 6x^2 + 8x$;

(i) $4x^3 - 12x^2 + 9x - 1$.

Sec. 8: Find all values t in the interval $[0, 2\pi]$ satisfying the given equation:

(a) $\tan t + 1 = 0$;

(b) $\cos t + \sqrt{3} = 0$;

(c) $\tan^2 t = \frac{1}{3}$;

(d) $\cos^2 t = \frac{3}{4}$;

(e) $|\tan t| = 1$;

(f) $|\sec t| = 1$;

(g) $2\sin 2t - \sqrt{2}\tan 2t = 0$;

(h) $\tan t - 3\cos t = 0$.

Sec. 9: Show the following identities are valid for all real number values of t:

(a) $\sin\left(t+\frac{3\pi}{2}\right) = -\cos t$;

(b) $\cos\left(t+\frac{3\pi}{2}\right) = \sin t$;

(c) $\sin\left(t+\frac{\pi}{2}\right) = \cos t$;

(d) $\cos\left(t+\frac{\pi}{2}\right) = -\sin t$;

(e) $\sin\left(\frac{3\pi}{2}-t\right) = -\cos t$;

(f) $\cos\left(\frac{3\pi}{2}-t\right) = -\sin t$.

Sec. 10: Show that the following identities are valid for almost all real number values of x (in each case, state precisely whether the validity is for ALL values of x; if not, explain carefully which x must be excluded from consideration):

(a) $(1-\cos^2 x)\sec^2 x = \tan^2 x$;

(b) $\cot x + \tan x = \sec x \csc x$;

(c) $\cos x = \sin x \sin 2x + \cos x \cos 2x$;

(d) $(\cos x - \sin x)^2 = 1 - \sin 2x$;

(e) $\tan x - \cot x = -2\cot 2x$;

(f) $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$;

(g) $\sec x - \cos x = \sin x \tan x$;

(h) $\cos x(\cot x + \tan x) = \csc x$;

(i) $\dfrac{\cos x}{1-\tan x} + \dfrac{\sin x}{1-\cot x} = \sin x + \cos x$

(j) $\dfrac{\sin x}{1-\cos x} = \cot x + \csc x$;

(k) $\cos^4 x - \sin^4 x = \cos 2x$.

Sec. 11: Find all values of x in the interval $[0, 2\pi]$ that satisfy the given equation:

(a) $\sin 2x = \sin x$;

(b) $\sin 2x = \cos x$;

(c) $2\sin^2 x + \cos x - 1 = 0$;

(d) $\cos^2 x - 3\sin x - 3 = 0$;

(e) $\tan x + \cot x = \dfrac{2}{\sin 2x}$;

(f) $2\cot^2 x + \csc^2 x - 2 = 0$;

(g) $\cos\dfrac{x}{3} = \dfrac{1}{2}$;

(h) $\sin 4x = -\dfrac{\sqrt{3}}{2}$;

(i) $2\sin^2 x - 3\sin x + 1 = 0$;

(j) $2\sin^2 x - \cos x - 1 = 0$;

(k) $\tan^3 x - 4\tan x = 0$;

(l) $2\tan x - 3\cot x = 0$;

(m) $\sec x - \tan x = 1$;

(n) $\sin 2x + \cos x = 0$.

Sec. 12: Find all real values of x that satisfy the given equation:

(a) $\log_x 4 = 2$;

(b) $1 - \ln(3x+2) = 0$;

(c) $\ln x + \ln(x-1) = \ln 2$;

(d) $\log_3(2x^2+17x) = 2$;

(e) $e^{2x} = 3^{x-4}$;

(f) $2\cdot 3^{-x} = 2^{3x}$;

(g) $\ln(2x-3) = 4$;

(h) $e^{3x-4} = 5$;

(i) $\ln(2x-1) + \ln(3x-2) = \ln 7$;

(j) $2\ln(x-1) - \ln(x-3) = 1$;

(k) $3^x \cdot 5^{x-2} = 3^{4x}$;

(l) $3\cdot 4^x = 2^{2x+1}$;

(m) $2e^x x^2 - e^x x = e^x$;