In[45]:= $r = \dfrac{k\,e}{1 + e\,\mathrm{Cos}[\theta[t]]}$ (* From Stein handout and Thomas 13.6. *)

Out[45]= $\dfrac{e\,k}{1 + e\,\mathrm{Cos}[\theta[t]]}$

v = {D[r, t], r D[θ[t], t]}   (* u_r, u_θ components *)

Out[38]= $\left\{\dfrac{e^2\,k\,\mathrm{Sin}[\theta[t]]\,\theta'[t]}{(1 + e\,\mathrm{Cos}[\theta[t]])^2},\ \dfrac{e\,k\,\theta'[t]}{1 + e\,\mathrm{Cos}[\theta[t]]}\right\}$

In[48]:= v0 = (Norm[v] /. {θ[t] → 0} // Simplify[#, Assumptions → {e > 0, k > 0, θ'[t] > 0}] &)
(* v_0 is the speed at the moment θ equals 0 *)

Out[48]= $\dfrac{e\,k\,\theta'[t]}{1 + e}$

In[44]:= a = {D[r, {t, 2}] - r D[θ[t], t]^2, r D[θ[t], {t, 2}] + 2 D[r, t] D[θ[t], t]}

Out[44]= $\left\{-\dfrac{e\,k\,\theta'[t]^2}{1 + e\,\mathrm{Cos}[\theta[t]]} + e\,k\left(\dfrac{e\,\mathrm{Cos}[\theta[t]]\,\theta'[t]^2}{(1 + e\,\mathrm{Cos}[\theta[t]])^2} + \dfrac{2\,e^2\,\mathrm{Sin}[\theta[t]]^2\,\theta'[t]^2}{(1 + e\,\mathrm{Cos}[\theta[t]])^3} + \dfrac{e\,\mathrm{Sin}[\theta[t]]\,\theta''[t]}{(1 + e\,\mathrm{Cos}[\theta[t]])^2}\right),\right.$
$\left.\dfrac{2\,e^2\,k\,\mathrm{Sin}[\theta[t]]\,\theta'[t]^2}{(1 + e\,\mathrm{Cos}[\theta[t]])^2} + \dfrac{e\,k\,\theta''[t]}{1 + e\,\mathrm{Cos}[\theta[t]]}\right\}$

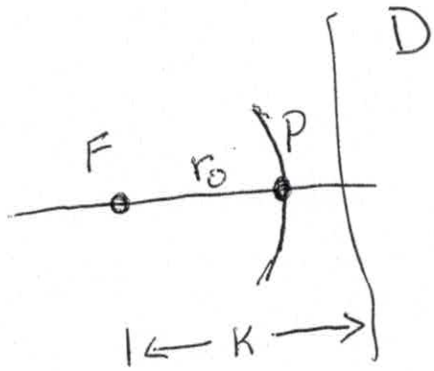
In[49]:= a0 = (Norm[a] /. {θ[t] → 0, θ''[t] → 0} // Simplify[#, Assumptions → {e > 0, k > 0, θ'[t] > 0}] &)
(* At the moment θ equals 0, r is at its minimum value,
and thus (since r^2 D[θ,t] is constant) D[θ,t] attains its maximum value,
and thus, at this moment, the second derivative of θ w.r.t. t must be 0. *)

Out[49]= $\dfrac{e\,k\,\theta'[t]^2}{(1 + e)^2}$

In[51]:= $\dfrac{v0^2}{a0}$

Out[51]= e k

(* Wow! That's nice. It follows that ke = $\frac{v_0^2}{a_0} = \frac{v_0^2}{GM/r_0^2} = \frac{r_0^2 v_0^2}{GM}$. *)



$FP = e\,PD$

At time "0":

$r_0 = e(K - r_0)$

So $Ke = r_0(1+e)$

$\quad \| $

$\dfrac{r_0^2 V_0^2}{GM}$

So $e = \dfrac{r_0 V_0^2}{GM} - 1$.