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(* Formula for computing connection forms for a field frame. *)
ω[ff1_, ff2_, ff3_] := Dt[{ff1, ff2, ff3}].Transpose[{ff1, ff2, ff3}] // FullSimplify // MatrixForm
(* Example 1 *)
e1 = {-2 (x y)/(x^2 + y^2), (x^2 - y^2)/(x^2 + y^2), 0};
e2 = {-(x^2 - y^2)/(x^2 + y^2), -2 (x y)/(x^2 + y^2), 0};
e3 = {0, 0, 1};
ω[e1, e2, e3]
```

$$\begin{pmatrix} 0 & \frac{2(-y\,Dt[x]+x\,Dt[y])}{x^2+y^2} & 0 \\ \frac{2\,y\,Dt[x]-2\,x\,Dt[y]}{x^2+y^2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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(* Example 2 [Spherical frame] *)
f1 = {Cos[ϕ] Cos[θ], Cos[ϕ] Sin[θ], Sin[ϕ]};
f2 = {-Sin[θ], Cos[θ], 0};
f3 = {-Sin[ϕ] Cos[θ], -Sin[ϕ] Sin[θ], Cos[ϕ]};
ω[f1, f2, f3]
```

$$\begin{pmatrix} 0 & Cos[\phi]\,Dt[\theta] & Dt[\phi] \\ -Cos[\phi]\,Dt[\theta] & 0 & Dt[\theta]\,Sin[\phi] \\ -Dt[\phi] & -Dt[\theta]\,Sin[\phi] & 0 \end{pmatrix}$$

```
g1 = {Cos[θ], Sin[θ], 0};
g2 = {-Sin[θ], Cos[θ], 0};
g3 = {0, 0, 1};
ω[g1, g2, g3]
```

$$\begin{pmatrix} 0 & Dt[\theta] & 0 \\ -Dt[\theta] & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$