11.1

14) $\displaystyle\sum_{k=3}^{10} \frac{k^k}{k!}$, $\displaystyle\sum_{i=0}^{7} \frac{(i+3)^{i+3}}{(i+3)!}$

17) set $k=n+3$. Then $n=-1$ when $k=2$ and $n=7$ when $k=10$

$$\sum_{k=2}^{10} \frac{k}{k^2+1} = \sum_{n=-1}^{7} \frac{n+3}{(n+3)^2+1} = \sum_{n=-1}^{7} \frac{n+3}{n^2+6n+10}$$

24) $\displaystyle\sum_{k=1}^{n} k(k^2-5) = \sum_{k=1}^{n} k^3 - 5\sum_{k=1}^{n} k = \left[\tfrac{1}{2}n(n+1)\right]^2 - \tfrac{5}{2}n(n+1)$

39) $\displaystyle\frac{62}{100} + \frac{1}{100}\sum_{k=1}^{\infty} \frac{45}{100^k} = \frac{62}{100} + \frac{1}{100}\left(\frac{45/100}{1-1/100}\right) = \frac{687}{1100}$

44) $\displaystyle\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = \sum_{k=0}^{\infty} (-x^2)^k = \sum_{k=0}^{\infty} (-1)^k x^{2k}$

47) $\displaystyle\frac{x}{1+x^2} = x\left[\frac{1}{1-(-x^2)}\right] = x\sum_{k=0}^{\infty} (-x^2)^k = \sum_{k=0}^{\infty} (-1)^k x^{2k+1}$

11.2

15) diverges; basic comparison with $\sum \frac{1}{k}$

25) diverges; limit comparison with $\sum \frac{1}{k}$

32) diverges; basic comparison with $\sum \frac{1}{\sqrt{k}}$, $\frac{2+\cos k}{\sqrt{k+1}} > \frac{1}{\sqrt{k}}$

34) $\displaystyle\sum \frac{n}{1+2^2+3^2+\dots+n^2} = \sum \frac{n}{1/6\, n(n+1)(n+2)}$ converges:

limit comparison with $\sum \frac{1}{n^2}$

40) $\int_{n+1}^{\infty} \frac{dx}{x^p} < \sum_{k=n+1}^{\infty} \frac{1}{k^p} < \int_{n}^{\infty} \frac{dx}{x^p}$

$$\Rightarrow \frac{1}{(p-1)(n+1)^{p-1}} < \sum_{k=1}^{\infty} \frac{1}{k^p} - \sum_{k=1}^{n} \frac{1}{k^p} < \frac{1}{(p-1)n^{p-1}}$$

47) a) If $a_k/b_k \to 0$, Then $a_k/b_k < 1$ for all $k \geq K$ for some K. But then $a_k < b_k$ for all $k \geq K$ and, since $\sum b_k$ converges, $\sum a_k$ converges (The basic comparison Theorem)

b) similar to a) except that this time we appeal to part (ii) of theorem 11.2.5.

c) $\sum a_k = \sum \frac{1}{k^2}$ converges, $\sum b_k = \sum \frac{1}{k^{3/2}}$ converges,

$$\frac{1/k^2}{1/k^{3/2}} = \frac{1}{\sqrt{k}} \to 0$$

$\sum a_k = \sum \frac{1}{k^2}$ converges, $\sum b_k = \sum \frac{1}{\sqrt{k}}$ diverges

$$\frac{1/k^2}{1/\sqrt{k}} = \frac{1}{k^{3/2}} \to 0$$

d) $\sum b_k = \sum \frac{1}{\sqrt{k}}$ diverges, $\sum a_k = \sum \frac{1}{k^2}$ converges

$$\frac{1/k^2}{1/\sqrt{k}} = 1/k^{3/2} \to 0$$

$\sum b_k = \sum \frac{1}{\sqrt{k}}$, $\sum a_k = \sum \frac{1}{k}$ diverges

$$\frac{1/k}{1/\sqrt{k}} = \frac{1}{\sqrt{k}} \to 0$$

48) Since $a_k/b_k \to \infty$, $b_k/a_k \to 0$ so this follows from before.

b) Follows from before

c) $\sum a_k = \sum \frac{1}{\sqrt{k}}$ diverges, $\sum b_k = \sum \frac{1}{k^2}$ converges

$$\frac{1/\sqrt{k}}{1/k^2} = k^{3/2} \to \infty$$

$\sum a_k = \sum \frac{1}{\sqrt{k}}$ diverges, $\sum b_k = \sum \frac{1}{k}$ diverges

$$\frac{1/\sqrt{k}}{1/k} = \sqrt{k} \to \infty$$

d) $\sum b_k = \sum \frac{1}{k^2}$ converges, $\sum a_k = \sum \frac{1}{k^{3/2}}$ converges

$$\frac{1/k^{3/2}}{1/k^2} = \sqrt{k} \to \infty$$

$\sum b_k = \sum \frac{1}{k^2}$ converges, $\sum a_k = \sum \frac{1}{\sqrt{k}}$ diverges,

$$\frac{1/\sqrt{k}}{1/k^2} = k^{3/2} \to \infty$$

50) since $0 < \left(a_k - \frac{1}{k}\right)^2 < a_k^2 + \frac{1}{k^2}$, $\sum \left(a_k - \frac{1}{k}\right)^2$ converges

by comparison with $\sum a_k^2 + \sum \frac{1}{k^2}$. But $\sum \left(a_k - \frac{1}{k}\right)^2 =$

$= \sum a_k^2 - 2\sum \frac{a_k}{k} + \sum \frac{1}{k^2}$ so $\sum \frac{a_k}{k}$ must converge

54) a) set $f(x) = 1/x$ in the proof of the integral test

b) $\sum_1^n s\frac{1}{k} > 100$ if $\log(n+1) > 100 \Rightarrow n+1 > e^{100} \approx 2.7 \times 10^{43}$

11.3

19) diverges; integral test: $\int_2^\infty \frac{1}{x}(\ln x)^{-1/2}\,dx = \lim_{b\to\infty}\left[2(\ln x)^{1/2}\right]_2^b \to \infty$

27) converges; basic comparison with $\sum \frac{1}{k^{3/2}}$

28) converges; ratio test: $\frac{(k+1)!}{1\cdot 2\cdot \ldots \cdot (2k+1)} \cdot \frac{1\cdot 3\cdot \ldots \cdot (2k-1)}{k!} = \frac{k+1}{2k+1} \to \frac{1}{2}$

30) converges; root test:

$$(a_k)^{1/k} = \frac{(2k+1)^2}{5k^2+1} \to \frac{4}{5}$$

37) $\frac{1}{2} + \frac{2}{3^2} + \frac{4}{4^3} + \frac{8}{5^4} + \ldots = \sum_{k=0}^{\infty} \frac{2^k}{(k+2)^{k+1}}$ converges;

root test: $(a_k)^{1/k} = \frac{2}{(k+2)^{1+1/k}} \to 0$

38) converges; ratio test (see #28)

45) The series $\sum_{k=0}^{\infty} \frac{k!}{k^k}$ converges. Therefore,

$$\lim_{k\to\infty} \frac{k!}{k^k} = 0 \text{ by Theorem 11.1.5.}$$