10.2

11) $\frac{n+(-1)^n}{n} = 1 + (-1)^n \frac{1}{n}$: not monotonic,

bounded below by 0 and above by $3/2$

27) $(-1)^{2n+1}\sqrt{n} = -\sqrt{n}$: decreasing, bounded above by -1 but not bounded below

41) For $n \geq 5$

$$\frac{a_{n+1}}{a_n} = \frac{5^{n+1}}{(n+1)!} \cdot \frac{n!}{5^n} = \frac{5}{n+1} < 1$$

and thus $a_{n+1} < a_n$. Sequence is not nonincreasing: $a_1 = 5 < 25/2 = a_2$

45) $a_1 = 1$ $a_2 = 1/2$ $a_3 = 1/6$ $a_4 = 1/24$ $a_5 = 1/120$ $a_6 = 1/720$ $a_n = \frac{1}{n!}$

49) $a_1 = 1$ $a_2 = 3$ $a_3 = 5$ $a_4 = 7$ $a_5 = 9$ $a_6 = 11$ $a_n = 2n-1$

57) First $a_1 = 2^1 - 1 = 1$. Next suppose $a_k = 2^k - 1$ for some $k \geq 1$.
Then $a_{k+1} = 2a_k + 1 = 2(2^k - 1) + 1 = 2^{k+1} - 1$

61) a) If $r=1$ then $S_n = n$ for $n = 1, 2, 3 \ldots$

b)
$$S_n = 1 + r + r^2 + \ldots + r^{n-1}$$
$$rS_n = r + r^2 + \ldots + r^n$$
$$S_n - rS_n = 1 - r^n$$
$$S_n = \frac{1-r^n}{1-r}, \quad r \neq 1$$

63) a) Let S_n denote the distance travelled between the nth and $(n+1)$st bounce. Then

$S_1 = 75 + 75 = 150$, $S_2 = \frac{3}{4}(75) + \frac{3}{4}(75) = 150\left(\frac{3}{4}\right) \ldots$ $S_n = 150\left(\frac{3}{4}\right)^{n-1}$

b) An object dropped from rest from a height h above the ground will hit the ground in $\frac{1}{4}\sqrt{h}$ seconds. Therefore it follows that the ball will be in the air

$$T_n = 2\left(\tfrac{1}{4}\right)\sqrt{\frac{s_n}{2}} = \frac{5\sqrt{3}}{2}\left(\frac{3}{4}\right)^{(n-1)/2} \quad \text{seconds}$$

$\boxed{10.3}$

27) Converges to $1/2$: $\frac{\sqrt{n+1}}{2\sqrt{n}} = \frac{1}{2}\sqrt{1+\frac{1}{n}} \to 1/2$

30) converges to $\sqrt{e}$: $\left(1+\frac{1}{n}\right)^{n/2} = \sqrt{\left(1+\frac{1}{n}\right)^n} \to \sqrt{e}$

35) $b < \sqrt[n]{a^n+b^n} = b\sqrt[n]{(a/b)^n+1} < b\sqrt[n]{2}$. Since $2^{1/n} \to 1$ as $n \to \infty$. it follows that $\sqrt[n]{a^n+b^n} \to b$ by the pinching Theorem

36) a) $-1 < r \leq 1$ b) $-1 < r < 1$

39) since $\left(1+\frac{1}{n}\right) \to 1$ and $\left(1+\frac{1}{n}\right)^n \to e$,

$$\left(1+\tfrac{1}{n}\right)^{n+1} = \left(1+\tfrac{1}{n}\right)^n\left(1+\tfrac{1}{n}\right) \to (e)(1) = e$$

40) a) If $k=j$ $a_n = \dfrac{\alpha_k + \alpha_{k-1}\cdot\frac{1}{n} + \dots + \alpha_0\frac{1}{n^k}}{\beta_k + \beta_{k-1}\cdot\frac{1}{n} + \dots + \beta_0\frac{1}{n^k}} \to \dfrac{\alpha_k}{\beta_k}$

b) If $k<j$ $a_n = \dfrac{\alpha_k + \alpha_{k-1}\cdot\frac{1}{n} + \dots + \alpha_0\cdot\frac{1}{n^k}}{\beta_j\cdot n^{j-k} + \beta_{j-1}\cdot n^{j-1-k} + \dots + \beta_0\cdot\frac{1}{n^k}} \to 0$

46) The converse is false. For example, let $a_n = (-1)^n$. Then $|a_n| \to 1$ but $\{a_n\}$ diverges.

56) $a_n = 1\cdot\frac{1}{2}\cdot\frac{2}{3}\cdot\dots\cdot\frac{n-1}{n} = \frac{1}{n}$, converges to 0.