MA370: Problem Set 3

Due: Wednesday, October 16 at 1pm

1. Show that L_k and L'_k obtained at the k-th step of Gaussian Elimination have the same structure. Here
$$L'_k = P_{m-1} \dots P_{k+1} L_k P_{k+1}^{-1} \dots P_{m-1}^{-1}.$$

2. Calculate by hand the LU decomposition of
$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix}$$
both with and without partial-pivoting. Then use both techniques to solve by hand
$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 11 \end{pmatrix}$$

3. TB: 20.1, 20.4, 21.6, 22.1, 23.1

4. MATLAB:

   - Generate a function `GaussPP.m` that takes as input an $n \times n$ matrix $\mathbf{A}$ and returns the matrices $\mathbf{P}, \mathbf{L}, \mathbf{U}$ such that $\mathbf{PA} = \mathbf{LU}$.
   - Create a driver that uses `GaussPP.m` to solve
   $$\begin{pmatrix} 2 & 1 & 5 & 1 \\ 3 & 2 & 11 & 1 \\ 2 & 8 & 7 & 3 \\ 7 & 4 & 4 & 2 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$$
   Please note the solution $\mathbf{x}$ and the matrices $\mathbf{P}, \mathbf{L}$, and $\mathbf{U}$.